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## Reduction to the Pole of Magnetic Anomalies Using Analytic Signal

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**Abstract:** Due to the dipolar nature of the geomagnetic field, magnetic anomalies observed anywhere rather than magnetic poles are asymmetric even when the causative body distribution is symmetric. This property complicates the interpretation of magnetic data. Reduction to the pole (RTP) is a technique that converts magnetic anomaly to symmetrical pattern which would have been observed with vertical magnetization. This technique usually is applied in frequency domain which has some disadvantages such as noise induction, necessity of using fixed inclination and declination throughout survey area and also unknown remanent magnetization that in many cases restrict its applicability. Analytic signal is a suitable quantity that can be calculated either in space or frequency domain and its amplitude is independent to magnetization direction. In this paper, analytic signal has been used as RTP operator and applied on the synthetic magnetic data and on the real magnetic data from an area in Shahrood region of Iran and results compared to conventional RTP operation. Results show that least difference is relevant to the causative body location and then analytic signal can be used as substituent method for conventional RTP.

**Key words:** Magnetic anomaly . inclination . declination . RTP . magnetization . analytic signal

### INTRODUCTION

Magnetic is a commonly used geophysical technique to identify and image subsurface targets in a wide range of applications from archeological site investigations to regional scale studies. Interpretation of magnetic anomalies is complicated process due to the dipolar nature of magnetic field, the superposition of multiple magnetic sources and presence of geological and cultural noises (such as noises due to pipe lines, power lines, railroads and etc). Furthermore an observed anomaly has asymmetric shape, when magnetization occurs in anywhere rather than magnetic poles. This makes a dipolar nature on magnetic field which causes a horizontal displacement between measured anomaly and exact body location. Pole reduction is an operator which takes magnetic anomalies and changes their asymmetric form to the symmetric and reproduces magnetic anomaly with vertical magnetization.

The frequency domain operator is [1]

$$A'(u,v) = \frac{A(u,v)}{(\sin I + \cos I \sin(D + \alpha))^2} \quad (1)$$

where  $A(u, v)$ , is the amplitude at frequencies  $(u, v)$ ,  $I$  and  $D$  are the geomagnetic inclination and declination respectively and  $\alpha = \tan^{-1}(v/u)$ . Implementation of this method in the frequency domain causes some problems.

It is unstable in low latitude, in the case that body has unknown remanent magnetization it gives incorrect results, induces synthetic noise to the data and lastly, frequency domain implementation of this technique, exigent that the inclination and declination values should be fixed entire the survey area.

There are several approaches that can be used to solve the latter problem. Cooper, (2005) uses Taylor series expansion in order to consider geomagnetic parameters variation throughout area of application of this filter [2]. This method is applied in space domain and includes computation of first and second order derivatives of standard reduction to the pole (RTP) about inclination and declination which in higher order of derivatives results tend to be noisy. Lu *et al.* (2003) reduced the dataset to the pole  $n$  times, where the dataset contains  $n$  datapoints. Therefore the inclination and declination can be varied at each grid point as required and only the response relevant to current point was backed up from each RTP operations [3]. This process repeated for all data points. The method is effective but necessity to considerable powerful computer restricts its application. Another technique is equivalent layer which can be used when inclination and declination vary over datasets [4]. In this case, an inversion stage that determines layer susceptibilities uses sources with different value of inclination and declination. Then the forwarded model is used to recalculate all layers with inclination set

to  $90^\circ$ . However, computational effort of inversion algorithm makes this technique inapplicable [5]. In cases that the variations of the field parameters are small, the study area can be divided to few sub-regions that in each sub-region, the fluctuations of inclination and declination can be negligible [6]. This method implies to easy performance but, because of requirement of large data storage, it is used rarely.

Obviously; the main purpose of magnetic data processing is simplification of acquired parameters from observed profiles and maps. One of these simplification approaches is creation a function which is independent to body magnetization direction and ambient geomagnetic parameters. These parameters are important when remanent magnetization is not negligible. Analytic signal is a quantity that includes this property and has been used to edge detection and depth estimation of magnetic bodies by several authors [7-10].

In this paper we introduce a new method for reduction to the pole of data which uses analytic signal. This method is applied on the synthetic magnetic data and also on real magnetic data from Shahrood area in north of Iran.

### ANALYTIC SIGNAL

The analytic signal or total gradient is formed through the combination of the horizontal and vertical gradients of the magnetic anomaly. The analytic signal has a form over causative body that depends on the locations of the body (horizontal coordinate and depth) but not on its magnetization direction. This quantity is defined as a complex function that its real component is horizontal gradient and its imaginary component is vertical gradient which can be proven that imaginary component is Hilbert transform of real component [11, 12].

Since the Hilbert transform plays an important role in the analytic signal computations, the brief review of it is followed:

The Hilbert transform of function  $f(x)$  is given by

$$F_1(x) = -\frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{f(x')}{x - x'} dx' \quad (2)$$

The inverse Hilbert transform is defined

$$f(x') = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{F_1(x)}{x' - x} dx \quad (3)$$

Fourier transform of both side of Eq. 2 are as following:

$$\mathfrak{F}(F_1) = i \operatorname{sgn} k \mathfrak{F}(f).$$

and

$$i = \sqrt{-1}$$

$$\operatorname{sgn}(k) = \begin{cases} 1 & \text{for } k > 0 \\ 0 & \text{for } k = 0 \\ -1 & \text{for } k < 0 \end{cases} \quad (4)$$

In Eq. 4  $k$  is Fourier wavenumber and  $\mathfrak{F}$  denote Fourier transform symbol.

Analytic signal of  $f(x)$  is defined as [12]:

$$a(x) = f(x) - i F_1(x) \quad (5)$$

Using Eq. 5 it is easy to derive the Fourier transform of analytic signal

$$\mathfrak{F}(a) = \mathfrak{F}(f)[1 + \operatorname{sgn} k] \quad (6)$$

where  $\mathfrak{F}(a)$  Fourier transform of analytic signal and  $\mathfrak{F}(f)$  is the Fourier transform of  $f(x)$ .

It follows that the analytic signal of function  $f(x)$  could be calculated in either of two ways: (1) by directly calculating the Hilbert transform of  $f(x)$ , as in Eq. 2 and then arrange it to  $f(x)$  as in Eq. 5; or (2) by Fourier transforming  $f(x)$ , setting to zero all values at  $k < 0$ , doubling all values at  $k > 0$  and inverse Fourier transforming the results.

Consider  $\varphi(x, y)$  be 2-D potential field that measured along  $x$ -axis, then the vertical and horizontal gradient of  $\varphi(x, y)$  with respect to  $x$  and  $z$  direction in Cartesian coordinate can be expressed as followed [13]:

$$\mathfrak{F}\left(\frac{\partial \varphi}{\partial x}\right) = i k \mathfrak{F}(\varphi) \quad (7)$$

$$\mathfrak{F}\left(\frac{\partial \varphi}{\partial z}\right) = |k| \mathfrak{F}(\varphi) \quad (8)$$

From Eq. 7 and Eq. 8 it can be shown that  $\frac{\partial \varphi}{\partial x}$  and

$\frac{\partial \varphi}{\partial z}$  are Hilbert transform pair. In particular, let select

$$f(x) = \frac{\partial \varphi}{\partial x}$$

and

$$F_1(x) = \frac{\partial \varphi}{\partial z}$$

substitution these relationships into Eq. 5 we can write the analytic signal form in the 2-D case as:

$$a(x,z) = \frac{\partial \varphi}{\partial x} + i \frac{\partial \varphi}{\partial z} \quad (9)$$

For the 3-D case, the analytic signal is given by

$$a(x,z) = \frac{\partial \varphi}{\partial x} + \frac{\partial \varphi}{\partial y} + i \frac{\partial \varphi}{\partial z} \quad (10)$$

The analytic signal has several well-known properties as followed

- Its real and imaginary components satisfy Cuchy-Reiman conditions.
- Its absolute value is symmetric rather than x-axis which is independent to body magnetization direction and ambient geomagnetic field and only is relevant to body location.
- This quantity can be employed to causative body depth estimation.
- Its maximum value lies over body directly.

Usually, the analytic signal is computed in the frequency domain using Fourier transform. In this approach the gradients of measured potential fields is found in the Fourier domain and combined as form of analytic signal. This tends to be noisy particularly, in case that the presence of man-made structures creates very high noise level to field data. For this reason, in this paper we applied the analytic signal in the space domain with calculating the horizontal gradient by finite difference method in the first step and then based on the role of Hilbert transform, the vertical derivative was created by Hilbert transform of horizontal gradient and formulated as the analytic signal. This approach does not enhance existence noises and its maximum value lies over body directly.

Finite difference theory can be used to achieve the horizontal derivatives of potential field data in the space domain and given by:

$$\frac{d\varphi(x,y)}{dx} = \frac{\varphi_{i+1,j} - \varphi_{i-1,j}}{2\Delta x} \quad (11)$$

$$\frac{d\varphi(x,y)}{dy} = \frac{\varphi_{i,j+1} - \varphi_{i,j-1}}{2\Delta y} \quad (12)$$

where  $\varphi_{i,j} = i, j = 1, 2, 3, \dots$  are measured magnetic data and  $\Delta x, \Delta y$  are distances between measured points in x and y direction respectively.

The amplitude of the analytic signal in the 3-D case given by:

$$|a(x,z)| = \sqrt{\left(\frac{\partial \varphi}{\partial x}\right)^2 + \left(\frac{\partial \varphi}{\partial y}\right)^2 + \left(\frac{\partial \varphi}{\partial z}\right)^2} \quad (13)$$

### APPLICATION TO THE SYNTHETIC MAGNETIC DATA

Figure 2 shows application of the analytic signal as RTP operator on synthetic magnetic data. The goal is to examine the applicability of the proposed method. The 2-D Polygonal cross-section of the model has been shown in Fig. 1. Model is related to an irregular shape body which lies on depth of 200m. The inclination and declination of body magnetization considered to be  $60^\circ$  and  $20^\circ$  respectively. Figure 2a shows the magnetic response of the model. Hot colors indicate the high values while cool colors represent low values. Figure 2b shows the standard RTP map of data in Fig. 2a in the frequency domain. Despite the considered model has no remanent magnetization and ambient geomagnetic parameters are known the dipolar nature of observed anomaly has not eliminated perfectly. Figure 2c shows the amplitude of the analytic signal map of data in Fig. 2a. Its maximum value lies over body and dipolar nature of the data has been removed completely. Figure 2d defines the difference between conventional RTP and the analytic signal which least difference is due to central part of image relevant to body location. It implies that the analytic signal can be used as an alternative method to conventional RTP without necessity of knowledge about magnetization parameters.

### APPLICATION TO THE REAL MAGNETIC DATA

Figure 3 shows the application of proposed method to the real magnetic data from Shahrood area in north of

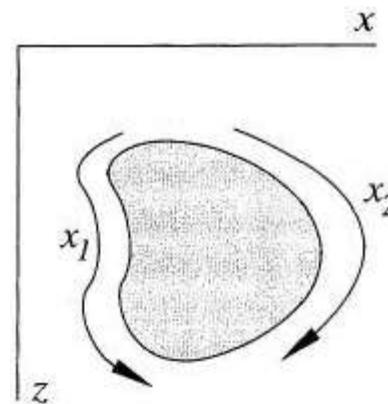


Fig. 1: 2-D irregular cross section used to produce magnetic anomaly [13]

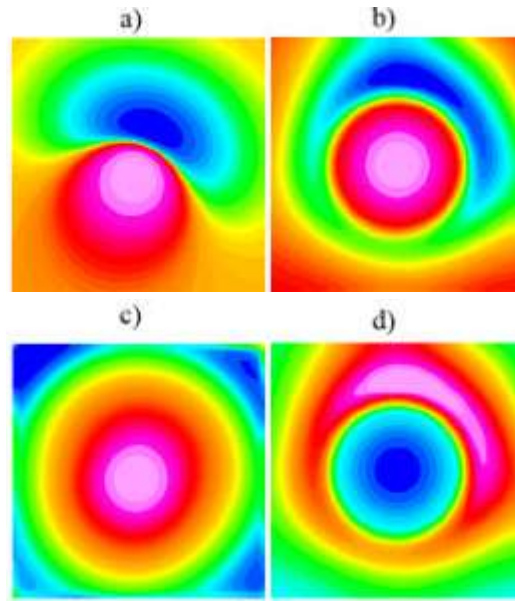


Fig. 2: Application to synthetic magnetic data: (a) Magnetic response from model with irregular cross section showed in Fig. 1. Depth to the top of the model is 200m and its inclination and declination are  $60^\circ$  and  $20^\circ$  respectively. Hot colors and cool colors are indicator of high and low intensity respectively. (b) Standard RTP map of data in (a) which the dipolar nature is presence. (c) The analytic signal map of data in (a). In this image the body dipolar nature are eliminated desirably. (d) Difference between data in Fig. 2 b and c. least difference is due to body region.

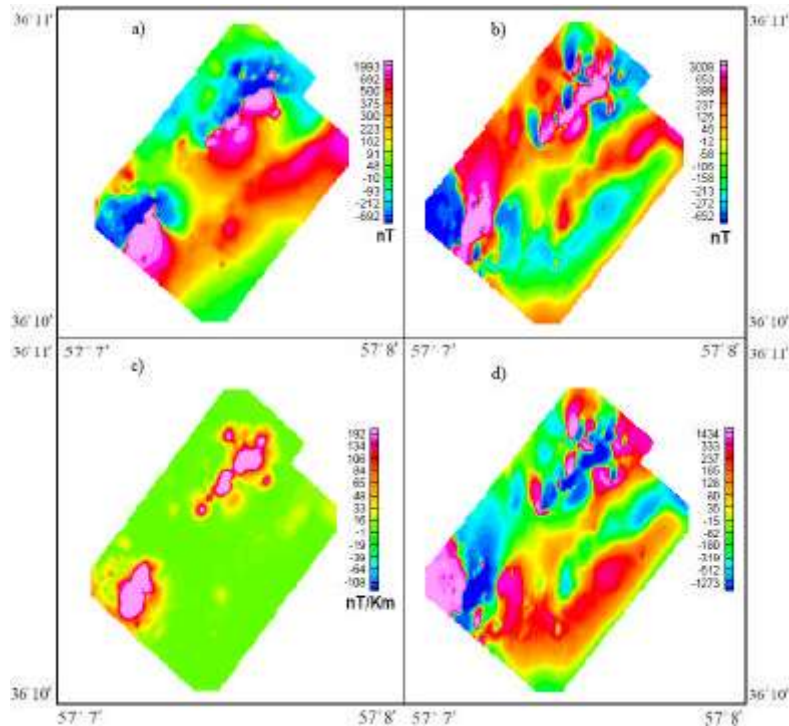


Fig. 3: Application to real magnetic data in Shahrood area: (a) Magnetic map of area. Inclination and declination values correspond to IGRF model are  $43^\circ$  and  $305^\circ$  respectively. (b) Standard RTP map of data in Fig. 3a. (c) Analytic signal map of data in Fig. 3a which enhances anomalies accurately. (d) Difference between data in Figs. 3 b and 3c

Iran. The magnetic data were collected using a proton-precession magnetometer with 0.01 nT accuracy. Magnetic measured points were selected on a 20m by 10m grid. The data were collected on the base of land survey corrected for diurnal variations. Totally, 840 data points along 28 profiles with length of 300m were acquired. Regional geomagnetic field was removed using IGRF model. The average of inclination and declination of geomagnetic field due to studied area are 43° and 3.5° respectively. The residual magnetic map of area under study has been shown in Fig. 3a. This image shows two specific dipolar anomalies with NE-SW trending. Figure 3b shows the conventional RTP map of data in Fig. 3a in the frequency domain. Since the body magnetization occurs in low latitude and has unknown remanent magnetization the conventional RTP possesses unstable results and dipolar nature preserved yet. Figure 3c shows the amplitude of the analytic signal map of magnetic data in Fig. 3a in the space domain. In this image dipolar nature of causative body has been removed desirably and noise enhancement is negligible. Figure 3d shows that the least difference between conventional RTP map and the analytic signal is related to body regions.

## CONCLUSION

Interpretation of the magnetic anomalies is a complicated process due to their dipolar nature. RTP technique eliminates the dipolar nature of magnetic anomalies and converts its asymmetric shape to symmetric shape. Providing the accurate results is subordinated to actual understanding of the magnetization parameters that in many cases this is inaccessible due to unknown remanent magnetization. Analytic signal is a quantity that is independent to magnetization characteristics and includes several approaches such as depth estimation and edge detection. In this paper this technique has been applied as RTP operator and applied on synthetic and real magnetic data from Shahrood area in the north of Iran. In observed magnetic map there are two dipolar anomalies presented which by application of the analytic signal this dipolar pattern has been removed completely.

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